Research Article

Stellar Aberration and Light-speed Constancy

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The distance traveled by a light ray from the vantage point of an observer on earth is analyzed in order to obtain a perspective on the phenomenon of stellar aberration. It is pointed out using vector addition that the latter distance is not the same as that traveled by the light relative to the sun. The required analysis in the case of the Fresnel light-drag experiment proceeds along a different track because it involves only one observer/detector making measurements under different conditions. In this situation the Relativistic Velocity Transformation (RVT) is applicable, as first noted by von Laue in 1907.

It is also pointed out the derivation of the RVT is based on Einstein’s light-speed constancy postulate, but is not dependent on the Lorentz transformation [LT]. This is an important observation since it has been shown in earlier work that two of its predictions, remote non-simultaneity and proportional time dilation, are mutually contradictory (Clock Puzzle). Instead, there is a different space-time transformation which also satisfies both of Einstein’s postulates of relativity which does not suffer from the same problem. The latter transformation, referred to as the Newton-Voigt transformation (NVT), eschews the LT space-time mixing characteristic entirely, and is thus consistent with Newtonian Simultaneity. The discussion concludes with a review of various experiments that have confirmed the existence of time dilation and have ultimately led to the development of the Global Positioning System (GPS). The asymmetry of time dilation, contrary to the LT symmetry prediction, in conjunction with Galileo’s Relativity Principle (RP), indicates that an amendment to the RP is in order. Accordingly, the laws of physics are assumed to be the same in each rest frame, but the units of time, distance and inertial mass on which they are based vary in a systematic manner (Universal Time-dilation Law) which depends uniquely on its velocity relative to a specific objective rest system (ORS) in each case.

Keywords: aberration of starlight at the zenith, Einstein’s light-speed constancy postulate, Galilean velocity transformation (GVT), relativistic velocity transformation (RVT), Lorentz transformation (LT), Newton-Voigt transformation (NVT)

Introduction

The phenomenon of stellar aberration refers to astronomical observations of the apparent movement of the positions of celestial objects at different times of the year. The first coherent explanation for this effect is credited to James Bradley. Writing in 1727, he ascribed it to the finite velocity of light and the motion of the earth relative to the sun, and he used the classical theory of motion to quantify his position. There was longstanding wide acceptance for his arguments, but they eventually met with considerable skepticism because they were thought to be incompatible with new experimental data obtained at the beginning of...
the next century. The latter results led to the development of numerous theories that posited the existence of an aether that was assumed to be essential to the true theory of the motion of light.

The matter came to a head in 1905 when Einstein published what has come to be known as the Special Theory of Relativity (STR) [1]. He rejected the need for an aether to explain the outstanding questions, but assumed instead that "light in a vacuum always moves with a definite velocity, independent of the velocity of the emitting body." This conclusion was in conflict with Bradley's explanation of stellar aberration, which assumed, in concert with the classical (Galilean) theory, that the speed of light emitted from the sun depends on the state of motion of an observer located on the earth's surface.

As a result of Einstein’s theory, a factor of \( \gamma = (1-v^2c^{-2})^{-0.5} \) was introduced into the classical expression for the angle of aberration of light emitted from the sun or any other source, whereby \( v \) is the speed of the observer relative to the light source and \( c \) is the speed of light in free space (299792458 m/s). For typical speeds of the earth relative to the sun, however, \( \gamma(v) \) differs from unity by on the order of only \( 10^{-8} \), and this difference is therefore too small to be confirmed in actual observations.

In order to understand the distinction between the above theories, it is important to consider the corresponding two velocity transformations, Galilean classical and Einsteinean relativistic, which are directly responsible for the disagreement about the necessity for including the above factor of \( \gamma \) in the aberration formula, and this topic will be taken up in the following discussion.

**Comparison of the Two Velocity Transformations**

The Galilean velocity transformation (GVT) in the form given in eqs. (1a-1c) involves two observers who are moving with speed \( v \) relative to each other along the arbitrarily chosen \( x, x' \) axis of the coordinate system. They each measure the velocity of the same object and their results are compared in the three equations of the transformation in terms of the respective components, \( u_x, u_y, u_z \) and \( u_x', u_y', u_z' \) (\( y \) and \( z \) are mutually perpendicular directions to \( x \)):

\[
\begin{align*}
    u_x &= (u_x' + v) \\
    u_y &= u_y' \\
    u_z &= u_z'.
\end{align*}
\]

A simple example serves to illustrate the relationships. If one of the observers finds that the object moves with speed \( u_x' = w \) along the \( x \) axis, his counterpart will measure the corresponding value to be \( u_x = v + w \) in the same direction. By contrast, according to the transformation they must agree on the values of any of the perpendicular components of the object’s velocity relative to their respective rest frames.

The above procedure can be conveniently described in terms of ordinary vector addition of the velocities. One vector connects the motion of the two observers, while another describes that of the object relative to one of them. The corresponding vector relating the motion of the object to the other observer is obtained by adding the above two vectors.

It is easy to see why the GVT is in conflict with Einstein's assumption of the constancy of light in free space. If the light pulse moves in the positive \( x \) direction with speed \( u_x = c \), eq. (1a) requires that \( u_x = c + v \). Yet, according to Einstein’s assumption, both \( u_x' \) and \( u_x \) should have the same value of \( c \). This is a clear contradiction between the two positions, since it amounts to saying that \( c = c + v \) regardless of the value of the relative speed \( v \) of the two observers, an obvious absurdity.

The Relativistic Velocity Transformation (RVT) is shown in eqs.(2a-2c). It contains the same variables as the GVT, but also makes use of two additional functions, \( \eta' = (1 + v^2/c^2)^{1/2} \) and \( \gamma = (1 - v^2/c^2)^{0.5} \).
The RVT is derived by assuming that space and time are mixed, a concept first introduced by Voigt in 1887 [2]. This position stands in stark contrast to the view of classical physicists such as Newton which holds that the two observers always agree on the amount of elapsed time in which measurements are made (\(\Delta t = \Delta t'\)). The RVT eliminates the "\(c = c + v\)" problem through the use of the \(\eta'\) function. If \(u_x = c\), then \(\eta' = (1 + c^2v^2)^{1/2}c\). As a result, in eq. (2a) \(u_x = (c + v)^{1/2}c\), in agreement with the light-speed constancy assumption. This is certainly not surprising, since the underlying condition in deriving the RVT is that for any choice of \(u_x', u_y', u_z'\) with a vector magnitude of \(c\), the corresponding result for \(u_x, u_y, u_z\) must also have the same magnitude, but with a generally different direction than the original vector. It should be noted that the RVT results cannot be obtained by vector addition, contrary to the situation with the GVT.

It is interesting to consider the above example from a different perspective, however, namely the distance traveled by each of the two observers and the object during a given elapsed time \(T\). The rest frame of the light source moves a distance \(vT\) relative to the location of the other observer (A), while the light pulse itself moves a distance of \(u_x'T = cT\) relative to the position of the source during the same time \(T\). As a result, the total distance traveled by the light pulse relative to observer \(A\) is the sum of these two values, \(cT + vT\). By the definition of speed as the ratio of distance traveled in unit time, this means that the speed of the light pulse relative to \(A\)'s position is \(u_x = (cT + vT)/T = c + v\). This value is in agreement with the prediction of the GVT, but contradicts that based on the RVT.

It also should be clear that the nature of the above result is the same for any velocity of the object. The distance traveled by the object relative to another observer can be deduced from the GVT solely on the basis of the corresponding distance measured by himself and his velocity relative to the latter. By contrast, the RVT can never obtain the correct result because it makes the false assumption that the two observers can measure the same velocity for a given object. On the contrary, the above analysis shows that the only way two observers can agree on the latter is if they are not moving relative to each other, i.e. are both stationary in the same rest frame (\(v = 0\)).

The above analysis is clearly relevant to the theoretical description of stellar aberration. Consider a light pulse emitted from the sun along the \(y\) axis of the coordinate system, so that in the usual notation, \(u_y' = -c\) and \(u_x' = u_z' = 0\). Assume that the sun moves along the \(x\) axis with speed \(v\) relative to the earth's location. According to the GVT of eqs. (1a-1c), the corresponding values for the observer on the earth's surface are \(u_x = v\) and \(u_y = c\). The RVT on the other hand predicts, since \(\eta' = 0\) in this case, \(u_x = v\) and \(u_y = \gamma'c\), a clearly different result. The tangent of the angle of aberration (tan \(\theta\)) between the trajectories of the light pulse observed on the earth and the sun, respectively, has a value of \(v/c\) for the GVT, but \(\gamma'v/c\) according to the RVT.

As in the abstract case discussed above, it is instructive to consider the distances traveled by the light in a given elapsed time \(T\) based solely on the assumed velocities of the light relative to the sun and the sun relative to the earth. Accordingly, one finds that the distance traveled by the light relative to the sun is \(cT\) in the \(y\) direction, while the distance the sun moves away from the earth is \(vT\). The distance the light has moved relative to the earth in this time is therefore equal to \((c^2 + v^2)^{1/2}T\). Note that neither the GVT nor the RVT has been used to obtain this result.

By the definition of speed as the ratio of distance traveled in a given time, the components of the velocity of the light relative to the earth are therefore \(u_x = v\) and \(u_y = c\), respectively. Moreover, the corresponding speed of the light is \((c^2 + v^2)^{1/2} = c\) and \(\tan \theta = v/c\), in perfect agreement with the GVT prediction, but in contradiction to the corresponding RVT result. The latter indicates by contrast that the speed of light relative to the earth is \((\gamma'c^2 + v^2)^{1/2} = c\), in agreement with the light-speed...
constancy requirement imposed by STR [1], but in conflict with the actual result obtained by consideration of the distances traveled by the light and the sun relative to the earth. As a consequence, it can be said conclusively that the RVT fails to give a satisfactory description of stellar aberration, whereas the GVT succeeds in this respect.

Moreover, a comprehensive review of the above arguments shows that corresponding discrepancies will occur whenever the RVT is applied to any situation in which the velocity results of two observers for the same object are compared. It fails because it necessarily disagrees with results obtained exclusively using vector addition to compute distances and velocities.

Required Conditions for Application of the RVT

It is clearly important to understand the conditions which require light-speed constancy in arriving at a coherent explanation for certain phenomena. The previous discussion has shown that this assumption has no validity for situations in which two observers in motion carry out measurements for the same object. The Fresnel light-drag experiment, on the other hand, is a concrete example of a case where light-speed constancy is required for its successful clarification, and so it is a good place to begin this discussion.

The experiment itself involves observations of the speed of light in transparent media. In the early 19th century, it was already clear that the value of the light speed varied when the speed of the medium v relative to the laboratory was increased. The measured value (c') was found to satisfy the formula given below (n is the refractive index of the medium):

\[ c' = \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right). \]  

(3)

If the value of n is changed to its free-space value (n=1), it is found that the v-dependence in eq. (3) disappears entirely, and one is led to conclude that \( c' = c(v) \) under this condition. This result is seen to be a verification of Einstein's [1] light-speed constancy assumption. The RVT of eqs. (2a-2c) leads to the same result for light moving in free space. Moreover, it also leads directly to eq. (3) when the light moves through a medium with refractive index n. This result was first obtained by von Laue in 1907 [3] and has been hailed as one of the first successes of Einstein's theory [4].

The derivation proceeds by assuming that \( u_s = c/n \) in eq. (2a). One then obtains in agreement with eq. (3):

\[ u_s = \eta \left(\frac{c}{n} + v\right) = \left(1 + \frac{c}{cn}\right) \left(\frac{c}{n} + v\right) = \]

\[ = \left(1 - \frac{v}{cn}\right) \left(\frac{c}{n} + v\right) = \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right). \]  

(4)

after making various approximations based on the condition that \( v << c \).

The crucial distinction in the Fresnel experiment is that there is only one observer in this case, as opposed to two in the example of stellar aberration. The quantities \( u_s \) and \( u_s' \) refer to the same observer making separate observations under two different conditions, namely v=0 and v\neq0. The assumption of light-speed constancy is then suggested by the special case for the free-space value of n=1, in which case \( u_s = u_s' = c \), as already discussed in connection with eq. (3). It is also clear that the GVT cannot be reasonably applied under this condition since it requires that two different observers are involved in making the speed determinations at the same time. In short, the range of application of the two velocity transformations is mutually exclusive. The RVT performs well for the Fresnel light-drag experiment, but not in the description of stellar aberration, whereas the opposite is the case for the GVT.

Another example for which the light-speed constancy
assumption is essential involves the acceleration of electrons in electromagnetic fields. The goal in this case is to cause an electron to attain faster-than-c speed. As in the Fresnel light-drag experiment, there is but one observer who performs measurements under two different conditions, before and after the field is applied. The assignments in the RVT of eqs. (2a-2c) of values of \( u_x, u_y, u_z \), on the one hand, and \( u_x', u_y' \) and \( u_z' \), on the other, are made on this basis. The assumption of light-speed constancy is justified because of the limiting case where the magnitudes of the two velocities each approach a value of c, i.e. one starts with the electron moving with a speed very close to c and ends up with a new velocity after application of the field with a magnitude which is only infinitesimally greater but is still less than c.

The RVT has the property of always relating velocities which are either both less than c or both equal to c or both greater than c \([5,6]\). It therefore has the possibility of reproducing the desired experimental relationship between the two electron velocities. By contrast, there is no way that the GVT of eqs. (1a-c) can be applied in this situation because that would require the existence of two distinct observers who are making their measurements at the same time. Once again, it is therefore seen that the ranges of application of the two transformations are mutually exclusive.

The Michelson-Morley experiment \([7]\) represents a different challenge entirely for the two theoretical approaches. Two light sources are involved, each of which emits a light pulse at the same time. The GVT is not applicable in this case because, although there are two observers/light sources, they are not moving with respect to one another \((v=0)\). The same holds true for the RVT.

What the Michelson-Morley experiment does show is that the speed of light relative to a given source is the same in all directions, something which is at least clearly consistent with Einstein's light-speed constancy assumption. It is interesting to see that the GVT can be applied if another observer is introduced who is moving relative to the laboratory. If his relative speed to the laboratory is \( v \), it follows on the basis of vector addition for the corresponding distances (as discussed in Sect. II) which have been traveled from his perspective that the measured speed of a given light pulse obtained by this observer would not be equal to c in either direction, nor would these two values be equal to each other. This observation emphasizes the fact that the light-speed constancy assumption only applies to velocities of light pulses measured relative to their respective source.

As a final example, the phenomenon of Thomas spin precession \([8]\) will be considered. This case has some similarities to that discussed above regarding attempts to accelerate an electron to faster-than-c speed. The focus in both cases is on the state of motion of the electron in two different situations, before and after application of a field, so the application of the GVT is ruled out in this case as well. The derivation of Thomas spin precession is different, however, in that it proceeds through the use of the Lorentz transformation (LT) rather than the RVT. The result is the following expression for the angular velocity \( \omega_T \) of the electron:

\[
\omega_T = \gamma c^2 \gamma^2 \left( \gamma + 1 \right) \mathbf{a} \cdot \mathbf{v},
\]

where \( \mathbf{v} \) and \( \mathbf{a} \), respectively, are the instantaneous velocity and acceleration of the electron at a given time.

The success of the Thomas derivation is widely taken to serve as a verification of the LT. The relationship between the RVT and the LT is not unique, however. Lorentz pointed out \([9]\) that light-speed constancy can be guaranteed in a more general version of the LT in which each of its four space-time equations is multiplied with a common factor \( \epsilon \). Einstein \([1]\) used \( \varphi \) to designate this quantity, and claimed that the only possible value for it that was also consistent with the RP is \( \varphi = 1 \). The discussion in the next section will show that there is another value for the \( \varphi/\epsilon \) function that is also consistent with both requirements, however. Furthermore, choosing the latter value is necessary in order to remove a fundamental
inconsistency in the LT.

The Clock Puzzle and Newtonian Simultaneity

The Lorentz transformation (LT) is often mistaken to be essential in derivations of relativistic phenomena, where in fact the RVT of eqs. (2a-2c) is wholly sufficient for this purpose. A case in point is starlight aberration, for which a relatively complicated derivation has been given based on the LT [10]. The same result is obtained using the RVT in fewer steps, as already discussed in Sect. II. However, it has also been shown that this result itself is incorrect and that the true value for the aberration angle can be obtained instead using the GVT exclusively.

Letting this point aside, however, it should be noted that there is a strong belief among physicists that the LT provides a completely reliable description of relativistic phenomena of all types. That this view is incorrect is easily shown in terms of what is referred to as the Clock Puzzle [11]. The LT makes two well-known predictions that need to be compared with one another. One is proportional time dilation, which is the slowing down of clocks due to their motion. Accordingly, observers in different rest frames must find that their measured elapsed times $\Delta t$ and $\Delta t'$ are related in the following way: $\Delta t = \gamma(v)\Delta t'$. The other is the alleged phenomenon of remote non-simultaneity, which calls for the same two observers to disagree as to whether two events occur simultaneously or not, e.g. $\Delta t' = 0$ and $\Delta t \neq 0$.

Both predictions come directly from consideration of the LT equation given below:

$$\Delta t = \gamma(\Delta t' + \nu c^{-2} \Delta x') = \gamma \eta^{-1} \Delta t'. \quad (6)$$

Remote non-simultaneity is seen to follow from eq. (6) whenever the relative speed $v$ of the two observers is non-zero at the same time that the spatial separation $\Delta x'$ of two events, such as lightning strikes on a train, is also not equal to zero. The prediction of proportional time dilation rules out this possibility, however, since this would require that multiplication of $\Delta t' = 0$, for simultaneous observation, by $\gamma(v)$ give a non-zero result for $\Delta t$. One has to reject the axiom of elementary algebra about multiplication with zero to believe otherwise [11,12].

It is clear from the above discussion that the LT prediction of remote non-simultaneity is a consequence of the mixing of space and time coordinates in eq. (6). It is a widely held view that this "space-time mixing" is essential in order to be consistent with Einstein’s light-speed constancy assumption. In this connection, however, it is important to recall that the rest frames described in both the LT and the RVT, as well as in the GVT, are inertial, i.e. there are no unbalanced external forces acting on them. According to Newton’s First Law of Motion (Law of Inertia), such a rest frame must proceed indefinitely with the same speed and direction. This raises a question about the properties of objects which are stationary in an inertial frame, particularly the rates of corresponding clocks. It seems unavoidable on the basis of the Law of Causality that they must also be constant for an indefinite period of time. This does not mean that the rates of all inertial clocks are equal, however, just as the velocities of different inertial rest frames do not have to be the same. What it does mean, however, is that the ratio of the rates of any two inertial clocks must be constant. As a result, one can reasonably conclude that elapsed times for a given event must always appear in the same proportion under these circumstances, i.e.

$$\Delta t = Q^{-1} \Delta t'; \quad (7)$$

where $Q'$ is the ratio of the corresponding rates of the two clocks. There is no space-time mixing in the above equation, and as a consequence, it is clear that it is also not consistent with remote non-simultaneity. This conclusion stands in full agreement with the long-held view of
classical physicists that space and time are completely separate entities. It will be referred to in the following as Newtonian Simultaneity.

Moreover, it is easy to find a replacement for the LT on this basis which is also consistent with Einstein’s light-speed constancy postulate [1]. One merely has to combine eq. (7) with the RVT of eqs. (2a-2c), using the definitions \( u_x' = \Delta x'/\Delta t', \) \( u_x = \Delta x/\Delta t, \) etc. [12-14]. The resulting set of equations, which will be referred to as the Newton-Voigt Transformation (NVT), is given below:

\[
\begin{align*}
\Delta x &= \eta' Q^{-1} (\Delta x' + v \Delta t') \\
\Delta y &= \eta' (\gamma Q')^{-1} \Delta y' \\
\Delta z &= \eta' (\gamma Q')^{-1} \Delta z' \\
\Delta t &= Q^{-1} \Delta t' \\
\end{align*}
\]

Voigt [2,15] was the first to use light-speed constancy in deriving a replacement for the GVT, and his space-time transformation is also consistent with the RVT [16].

It is easy to show that the inverse set of equations for the NVT can be obtained by simply reversing the sign of \( v \) and interchanging the meanings of the two sets of coordinates. For this purpose, it is necessary to define the corresponding proportionality factor in the inverse of eqs. (8a-d) to be \( Q' = Q^{-1} \). From a physical point of view, this condition simply reflects the fact that the two proportional factors have the reciprocal relationship expected for comparison of elapsed times from the different vantage points of the two rest frames. In addition, the fact that the inverse set of equations can be obtained in this way (Galilean inversion) is proof that the RP is satisfied in this definition as well.

The NVT corresponds to a factor of \( \epsilon' = \gamma' (\gamma Q')^{-1} \) in the general form of the Lorentz Transformation mentioned above. It is important to note that the corresponding factor \( \epsilon = \gamma (\gamma Q)^{-1} \) in the inverse set of equations is the reciprocal of \( \epsilon' \). This is a necessary relationship for satisfying the RP as well as the light-speed constancy condition (note that \( \eta' = \gamma^2 \) in the RVT [13,14], and as already mentioned, \( QQ' = 1 \) in the NVT).

The \( \epsilon' = 1 \) relationship for the NVT has an important bearing on the derivation of eq. (5). In order to arrive at the latter result for Thomas spin precession it is necessary to apply "Lorentz boosts" in both the forward and reverse directions of the electron’s motion. Replacing the LT by the NVT in the derivation means that the result for \( \omega_t \) is multiplied by a factor of \( \epsilon \epsilon' \) and therefore is not changed at all relative to the value in eq. (5). There is thus no reason to prefer the LT over the NVT in this respect, whereby the NVT has the overall advantage of not being self-contradictory with regard to the question of remote non-simultaneity.

The Transverse Doppler Effect

One of the ancillary consequences of the time-dilation phenomenon was Einstein’s prediction of a red shift in the frequency of light emitted from a moving source. This became known as the Transverse Doppler Effect (TDE) because it would at least theoretically be observed even when the light source was moving in a perpendicular direction to the line-of-sight of the observer. Quantitatively, it is expected that the frequency of light would be decreased by a factor of \( \gamma(v) = 1 + 0.5c^2v^2 \) relative to its standard value.

Since the conventional Doppler effect is proportional to \( v/c \) but also has an angular dependence of \( \cos \chi \) (\( \chi \) is the angle of approach of the light waves to the observer), it is clear that the TDE can be observed to sufficient accuracy only if one or both of the following conditions are met: eliminate the angular dependence by having the motion be transverse (\( \chi = \pi/2 \)) or cancel out the \( v/c \) dependence by carrying out two experiments in which the speed of the source is aligned in opposing directions and averaging the result. Ives and Stilwell [17] succeeded in carrying out the
necessary experiment by using the latter approach for an accelerated H-atom light source, but they did so by measuring the wavelength \( \lambda \) of the emitted light rather than the frequency. To a sufficiently good approximation, they were able to verify Einstein’s prediction by measuring the averaged wavelength \( \lambda_{av} \) to be equal to \( \gamma \lambda_0 \) (\( \lambda_0 \) is the standard value of for the light source at rest in the laboratory). A more accurate version of this experiment was carried out later [18] which removed any doubt about the fact that the wavelength of light increases upon acceleration. This result was simply taken to be equivalent to the predicted decrease in frequency, i.e. \( v = v_0 \gamma \), by virtue of the assumption of light-speed constancy in both rest frames (\( \gamma \lambda = v_0 \lambda_0 = c \)).

If one concentrates instead on the actual measurement of the speed of light in the two rest frames, however, one is confronted with the same situation discussed in Sect. II. If the light moves with speed \( c \) relative to the source, it must travel a distance of \( cT \) in a given time \( T \). Meanwhile, the light source has moved a distance of \( \sqrt{v}T \) relative to the laboratory rest frame. Thus, the speed of light relative to the laboratory is \( (cT + \sqrt{v}T) / T = \sqrt{v} + c \). This value is in agreement with the GVT prediction, but not with the RVT, since the latter predicts that the light speed relative to the laboratory must have a value of \( c \).

The finding of a red shift in the Ives-Stilwell experiment has been considered to be a major success of the LT. This view overlooks the fact that the same theory predicts that distances should be contracted in the moving rest frame (FitzGerald-Lorentz length contraction) and is therefore contradicted by the observed increase in wavelength. This failure is consistent with the result discussed in Sect. IV (Clock Puzzle) which demonstrates that the LT is self-contradictory with regard to its dual predictions of remote non-simultaneity and proportional time dilation.

The results of the TDE experiment are also in quantitative agreement with studies carried out for accelerated muons [19]. In that case, it was found that the lifetime of the muons increased by a factor of \( \gamma(v) \). This result was also based directly on length measurements, however. The average decay length of the muons was found to increase in direct proportion to \( \gamma(v) v \), whereby the expectation based on a constant lifetime would simply be that this quantity increases in direct proportion to \( v \) itself.

The discovery of the Mössbauer effect enabled Hay et al. [20] to carry out a different version of the Ives-Stilwell experiment in which frequencies are measured directly rather than wavelengths. In this case a high-speed rotor was used on which both an \(^{57}\)Fe x-ray source and detector were mounted at different positions along its radius. This arrangement assured that the radiation was detected transverse to the source, but more significantly, by locating the detector at the rim of the rotor, it provided a test of another important prediction of Einstein’s theory [1], namely that the TDE is a symmetric phenomenon. This was another direct consequence of the LT. The claim was made that the rates of clocks in both rest frames would slow down relative to each other, something which led to a great deal of consternation among physicists in general. In the present case, this meant that a red shift would be observed independent of whether the source was moving faster or slower than the detector. Placing the latter on the rim of the rotor guaranteed that it would be moving faster than the source, which was therefore the opposite situation to that in the Ives-Stilwell experiment [17,18], in which case the light source was accelerated relative to the laboratory rest frame.

The result of the Hay et al. experiment was therefore quite surprising [21], because it found that the x-ray frequency was blue-shifted in this arrangement. Two other versions of the experiment were carried out [22,23], so there is no doubt that this LT prediction is also incorrect. This finding is not at all surprising when seen in the context of the incompatibility of the LT predictions of remote non-simultaneity and proportional time dilation [11, 24, 25]. At the same time, the observation of asymmetric time dilation is quite consistent with both Newtonian Simultaneity and the NVT of eqs. (8a-8d).

There is another important benefit of the Hay et al. rotor experiment, however. The fact that the frequency of light is inversely proportional to \( \gamma(v) \) for any speed \( v \) of the light source, while the Ives-Stilwell experiment indicates that
the corresponding wavelength of light is directly proportional to the same quantity on a quite general basis, leads to the unequivocal conclusion that the speed of light in free space is indeed the same in all rest frames. Since wavelengths can only be measured to within an accuracy of one part in \(10^8\), it was decided to define the value of the speed of light to have a fixed value [26] which is given to eight significant figures (299792458 ms\(^{-1}\)).

**GPS and the Uniform Scaling of Physical Properties**

Attempts to better understand the nature of time dilation were given a big boost with the advent of atomic dilution. It became possible to measure elapsed times to quite high accuracy without making use of the transmission of light signals. Hafele and Keating [27,28] placed atomic clocks onboard commercial airplanes traveling in opposite directions around the earth. When the clocks were returned to the airport of origin after their respective flights, it was found that those flying in the easterly direction had significantly less elapsed time than their counterparts left behind at the airport, while those in turn had less elapsed time than the clocks which had made the circumnavigation in the westerly direction.

The observed asymmetry in clock rates was explained on a quantitative basis in terms of the velocities of the clocks relative to the earth’s center of mass (ECM), or rest frame of the “non-rotating polar axis” in the phrase used by the authors. The trajectories of the flights were broken down into small segments during which time the speed of each plane was considered to be constant. For all practical purposes, the fact that this could be successfully assumed in their calculations demonstrated that the planes were effectively inertial systems and therefore subject to the conditions generally thought to be essential for application of STR.

This state of affairs was completely inconsistent with the explanation for the asymmetry that had been given earlier in connection with the rotor experiments [20-23]. It had been claimed that it was the high degree of rotational acceleration that was responsible for the LT’s inability to predict the blue-shifted frequency that was observed in this study. Einstein’s Equivalence Principle [29] had been invoked to argue that a gravitational field had been generated at the position of the detector as a consequence of the rotation and this had caused the decreased frequency of the clock associated with the x-ray detector at the rim [22], which in turn led to the measured increase of frequency relative to the value emitted at the source. Another clear indication of the error in this analysis was the finding that the clock rates in the Hafele-Keating [27,28] experiment were directly affected by the actual differences in gravitational potential of the airplanes relative to the ground location. The authors were able to account for this effect on a quantitative basis by taking account of the gravitational red-shift formula predicted by Einstein in his 1907 paper [29].

After correcting for the above gravitational effect, Hafele-Keating were able to fit their time results in terms of the simple proportionality relationship given below:

\[
\Delta t' \gamma(v') = \Delta t \gamma(v) \tag{9}
\]

The speeds \(v\) and \(v'\) of the clocks in this formula are taken relative to the ECM. The indication is that the rate of a given clock was always inversely proportional to this speed during the entire course of the flights. There is thus a clear connection between these results in eq. (9) and the theoretical relationship given in eq. (7) based on Newtonian Simultaneity. In particular, it enables one to compute the value of \(Q'\) in the NVT of eqs. (8a-8d) as follows:

\[
Q' = \frac{\Delta t'}{\Delta t} \frac{\gamma(v)}{\gamma(v')} \tag{10}
\]

Moreover, the corresponding value of \(Q\) in the inverse transformation can also be evaluated in a consistent manner as
This reciprocal relationship is naturally expected, since it corresponds to a role reversal for the respective observers in the two rest frames.

It also can be noted that eq. (10) is also directly applicable to the results obtained in the rotor studies [20-23]. In this case the speeds in the \( \gamma \) factors must be taken relative to the laboratory rest frame. More generally, eq. (10) appears to be applicable to any situation in which an object undergoes acceleration as a result of the application of some external force. In that case, the speeds \( \nu \) and \( \nu' \) of the clocks are to be taken relative to the rest frame in which the force is applied. For this reason, it is appropriate to refer to eq. (10) as the Universal Time-dilation Law (UTDL) [30]. Another useful term is "objective rest system" or ORS [31], which designates the rest frame from which to evaluate the speeds in the associated \( \gamma \) factors in the UTDL. It is the ECM in the case of the circumnavigating atomic clock experiment [27,28], the rotor axis in the Hay et al. study [20-23] and the point of the applied force in the general case.

The experiments with circumnavigating clocks had a decisive effect on the development of the Global Positioning System (GPS). The key element in the distance determinations in this engineering feat is the use of atomic clocks to measure the elapsed time \( \Delta t \) required for a given light pulse to travel between an orbiting satellite in a known location and a receiver on the ground [32,33]. The result is then multiplied with \( c \) to obtain the corresponding value of the instantaneous distance, thereby making use of the known constancy of light in free space. To obtain an accurate measurement of \( \Delta t \) clearly requires that the two clocks in question are running at the same rate. Because of the effect of time dilation on the satellite clock, the desired equality can only be obtained by adjusting the rate of one of the clocks, however. To achieve this end, one has to know the value of the proportionality constant in eq. (7), which in turn can be evaluated using the UTDL and eq. (10). An additional correction needs to be made to account for the difference in gravitational potential of the two clocks [32,33], but this can be accurately done by using of Einstein's red-shift formula, also as demonstrated by the Hafele-Keating results. A "pre-correction" procedure [32] is used for this purpose according to which the rate of the satellite clock is adjusted prior to launch. This procedure assumes a perfectly circular orbit for the satellite. A real-time correction could be made in its place by simply multiplying the elapsed time on the satellite clock by the appropriate factor for a given altitude and orbital speed [34,35].

A particularly simple way to express the results of time dilation is to assume that the unit of time [36] in a given rest frame is changed upon linear acceleration, i.e. by a change in its velocity relative to the relevant ORS [31]. The proportionality factor \( Q' \) in eq. (7) can be looked upon as a conversion factor between the units of time in the two rest frames of interest. Accordingly, the conversion factor \( Q \) in the opposite direction has a value of \( 1/Q' \), i.e. the reciprocal of \( Q' \). This approach is perfectly consistent with the RP. In other words, the laws of physics are the same in each inertial system, but the units in which they are expressed can and do vary from one rest frame to another [12,37].

Moreover, other physical units are also affected in an analogous manner. Because of light-speed constancy, however, the unit of relative speed between two objects is unchanged by acceleration [38]. If this were not the case it would be possible to demonstrate a failure of the RP simply by comparing the value of a known relative speed with \( c \). Recognition of this point has consequences with regard to the unit of distance. Since the meter is defined in terms of the speed of light [26], it follows that any distance \( L = c\Delta t \) measured in one rest frame must have a value in the other of \( L' = c\Delta t' = (Q')^{-1}c\Delta t = (Q')^{-1}L \). Thus, one can have an equivalent relationship to the UTDL of eq. (9) for distances, and the conversion factor in this case is exactly the same as for time \( (Q')^{-1} \).

Experiment [39] has also shown that inertial masses \( m \) increase with acceleration in the same proportion as lifetimes and distances, so the conversion factor between
rest frames is also equal to \( Q^{-1} \) in this case. All other conversion factors for different physical properties can be obtained as integral powers of \( Q' \) on the basis of the composition of each quantity in terms of the fundamental units of inertial mass, distance and time. For example, frequency has the conversion factor of \( Q' \) since it is the reciprocal of that for the period of the corresponding clock. The factor for angular momentum \( l = m v r \) is \( Q'^{-2} \) (and therefore also for Planck’s constant \( \hbar \) since the latter has units of angular momentum) because the sum of the powers for the three components properties is -1+0+-1=-2. This choice is consistent with Planck’s radiation law \( (E=\hbar \nu) \), since energy \( E \) scales as \( Q'^{-1} \), while the corresponding frequency scales as \( Q' \). It is even possible to have analogous conversion factors for electromagnetic quantities [40,41]. An analogous scheme [42] is also possible for gravitational scale factors, but it is based on a different fundamental quantity than \( Q' \).

**Conclusion**

When a light source moves with speed \( v \) relative to an observer and emits a light pulse with speed \( c \) in the same direction in free space, by definition, the latter travels a distance \( cT \) in a given time \( T \) while the source itself has moved an additional distance of \( vT \) from the observer’s vantage point. This means that the light pulse is now located at a total distance of \( vT+cT \) away from the observer, which in turn means, also by definition, that the speed \( w \) of the light pulse relative to him is \( w = (vT+cT)/T = v+c \). The fact is that \( w \neq c \) stands in clear opposition to the prediction of \( w=c \) that results from application of the RVT of eqs. (2a-c), but it is in quantitative agreement with the classical GVT given in eqs. (1a-c). This state of affairs is by no means an exception, but rather occurs for any direction and speed that a light pulse might move relative to a given observer.

The analogous argument holds for any object moving with a speed less than \( c \) relative to one observer, i.e. the GVT will always give the correct velocity of the object relative to any other observer, while the RVT must necessarily give a different and therefore incorrect value for the same quantity. Another way to look at this is to see that any two such *distances* must be mathematically combined using conventional *vector addition*, which operation is completely consistent with the GVT, but not with the RVT.

When one applies this result to the case of stellar aberration, it becomes clear that the explanation for this phenomenon in standard relativity theory (STR [1]) is also incorrect, since it assumes that the distance a light wave travels from the sun \( (cT) \) in a given time \( T \) must be the same as that relative to an observer on the earth who naturally is moving relative to the sun. The tangent of the aberration angle \( \theta \) between the two directions is thus equal to \( v/c \), not the value of \( \gamma(v)v/c \) widely believed to be the case by the great majority of physicists at the present time, but rather the value predicted by Bradley in 1727.

Further consideration shows that Einstein’s light-speed constancy assumption, and the RVT on which it has been derived, is only applicable under fundamentally different circumstances. Instead of two observers and one light pulse/object, there must be *only a single observer* who is making his measurements under two different conditions and therefore at different times. A prime example for this is the Fresnel light-drag experiment which was first carried out in the early 18th century. In that case, the same observer measures the speed of light in a transparent medium such as water to have different values depending on the speed \( v \) of the medium relative to the observer. The GVT is not applicable in this case, whereas it was shown by von Laue in 1907 that the RVT is. The same holds true for other famous successes of Einstein’s version of relativity theory such as the inability to propel electrons to greater-than-\( c \) speed by successive applications of electromagnetic fields and the prediction of electron spin precession by Thomas 20 years later. In both cases, it is the application of different levels of force on a single object at different times which is responsible for the effect. The explanation for the null result of the Michelson-Morley experiment lies entirely within the purview of the light-speed constancy assumption and does not require
application of either the RVT or the GVT.

Another aspect of this discussion is the role the Lorentz transformation (LT) plays in the description of the above effects. It is widely held to be essential for the derivation of the RVT, but this is not the case. This is a critical observation since it is easy to show that the LT itself is self-contradictory and therefore invalid. It predicts that both proportional time dilation and remote non-simultaneity can occur at the same time, but this would require (Clock Puzzle) that two numbers/time differences ($\Delta t$ and $\Delta t'$) can always occur in the same proportion, and yet one of them can be zero (for simultaneous observation) while the other is not. This is clearly impossible because it violates the axiom of elementary algebra which states that multiplication of a number by zero must give a product of zero as well.

Consideration of the LT shows that its space-time mixing characteristic in eq.(4) is directly responsible for the contradiction. On the contrary, Newton’s First Law of Motion indicates that an inertial clock will always have the same rate and that the ratio of the rates of any two of them must itself therefore be a constant as well. This condition is expressed in the proportionality relation of eq. (7), which is clearly inconsistent with the existence of remote non-simultaneity. By combining the latter with the RVT, one obtains the alternative version of the LT shown in eqs. (8a-8d), which has been referred to as the Newton-Voigt transformation (NVT). It also satisfies both of Einstein’s relativity postulates, and does so without coming into any conflict with space-time relations. It denies the widely held view of the existence of space-time mixing and claims instead that there is no such thing as remote non-simultaneity. In the present context, one of its main characteristics is that the RVT can be derived from it in the same way as the LT does, i.e. by simple division of the various distance components by the corresponding elapsed time.

The speed of light in the Ives-Stilwell experiment relative to the laboratory is also not equal to $c$ and is determined quantitatively using the classical GVT. The experiment itself served as a verification of one of Einstein’s most famous predictions, the transverse Doppler effect (TDE). It also contradicted another prediction, however, namely FitzGerald-Lorentz length contraction, since it demonstrated that the wavelength of light can increase with acceleration of the source, showing once again that the LT, from which the effect has been derived, is not a reliable component of relativity theory. Later measurements with an x-ray source and detector mounted on a high-speed rotor showed that the frequency of light can be blue-shifted relative to the laboratory observer. This result serves as proof that the effects of time dilation are not symmetric, as was first pointed out by Sherwin [21], again in opposition to Einstein’s prediction based on the LT.

The Hafele-Keating experiment with circumnavigating atomic clocks was quite instructive. It showed that the asymmetry of time dilation was consistent with their finding that elapsed times measured with eastward-flying clocks were smaller than for their westerly counterparts. They also showed that gravitational effects on the clock rates were completely separate from those of time dilation. They were able to fit their timing results to a simple formula which indicated that the rate of a given clock, after separation of the associated gravitational effect (red shift), is inversely proportional to $\gamma (v)$, where $v$ is the speed of the clock relative to the ECM. The same formula holds for the rotor experiment except that the reference for the required speed determinations is the laboratory rest frame in this case. On this basis, it is possible to compute the value of the proportionality constant in the Newtonian Simultaneity expression of eq. (7), which in turn appears in each of the four equations of the NVT.

The Hafele-Keating results had a significant impact on the development of the GPS navigation system, since they showed how atomic clocks carried onboard satellites can be adjusted quantitatively to run at the same rate as their counterparts on the earth’s surface. This adjustment was essential for achieving the key goal of obtaining accurate measurements of the distance between a satellite of known position and the earth. By measuring the elapsed time it takes for a light signal to pass between them, the corresponding distance can be obtained by simply
multiplying this value with c, but this can only be done with sufficient accuracy if the clocks at both termini run at the same rate. The fact that this procedure works well throughout the world on an everyday basis shows that time dilation is a real effect, and that the necessary adjustment of clock rates on the GPS satellites can be done quantitatively on the basis of eq. (7). It also shows that Newtonian Simultaneity is real since it would be useless to make such an adjustment if events which are simultaneous on the satellite are not also simultaneous elsewhere in the universe.

The verification of asymmetric time dilation should be seen in the context of Galileo’s RP. It shows that although the laws of physics are the same in each rest frame, the rates of clocks are not. Further consideration indicates that the proportional factor Q’ applies to measurements of inertial mass and distance as well as for elapsed times. There is a \textit{uniform scaling} of physical properties as a result of linear acceleration, but because of this uniformity, it is impossible for an observer to notice these changes on the basis of his \textit{in situ} measurements alone (Galileo’s ship). These considerations indicate that the RP must be amended as follows: The laws of physics are the same in each rest frame, but the units on which they are based vary in a systematic manner from one system to another.

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