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Research Article

The Newton-Voigt Space-time Transformation

Robert J. Buenker[1]

[1]*Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany*

The cornerstone of the Special Theory of Relativity (STR) is the Lorentz transformation (LT). Its forerunner was given by W. Voigt in 1887 shortly after the publication of the Michelson-Morley interferometer experiment. It was based on his conjecture that the classical (Galilean) space-time transformation needed to be amended so that it would be consistent with experimental findings that indicate that the speed of light in free space is independent of the state of motion of the light source. In order to accomplish this objective, Voigt introduced for the first time the concept of space-time mixing, which has since become a doctrine of theoretical physicists. However, the Voigt transformation (VT) proved to be deficient because of its inability to adhere to the prescriptions of Galileo's Relativity Principle (RP).

A decade later, Larmor modified the VT to remove this inconsistency and the resulting set of equations has since become known as the LT. Although the LT satisfies both the RP and the light-speed constancy requirement, which have subsequently been referred to as Einstein's two postulates of relativity, it nonetheless also has a clear deficiency itself since it leads directly to two predictions that are mutually exclusive of one another, namely proportional time dilation and remote non-simultaneity. It has previously gone unnoticed by the physics community that an axiom of elementary algebra needs to be ignored in order to justify the co-existence of both of the above effects.

The Newton-Voigt transformation (NVT) also satisfies both of Einstein's postulates, but avoids any conflict between clock-rate predictions. It does so by invoking Newtonian Simultaneity, whereby the rates of any two inertial clocks, which necessarily have constant but different rates, must always be strictly proportional to one another as long as no unbalanced external force is applied to them. As a consequence, the apparent necessity of the mixing of space and time that Voigt foresaw is eliminated. Furthermore, the NVT is seen to be consistent with an exclusively objective view of the measurement process, something that is ruled out by both the VT and the LT.

Keywords: space-time transformation, Lorentz transformation (LT), Voigt transformation (VT), Newton-Voigt transformation (NVT).

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Introduction

After the results of the Michelson-Morley interferometer experiment [1] had appeared in 1887, W. Voigt, a German scientist, made a suggestion [2] that had a lasting effect on the way physicists view the relationship between space and time. A wide-ranging discussion [3] had started with the Fizeau/Fresnel light-drag experiment. It showed that light

* Correspondence: Robert J. Buenker, Fachbereich C-Mathematik und Naturwissenschaften, Bergische Universität Wuppertal, Gaussstr. 20, D-42097 Wuppertal, Germany.
E-mail: rjbuenker@gmail.com

is slowed as it moves through a transparent medium. By extrapolation of the medium's refractive index n to a unit value, however, it also indicated that in the limit of free space the observed light speed c(v) in the laboratory should be completely independent of the speed v of the medium, i.e. [$c(v) = c$]. This result is not consistent with the classical (Galilean) transformation given below in eq. (1):

$$\Delta t' = \Delta t \quad (1a)$$

$$\Delta x' = \Delta x - v\Delta t \quad (1b)$$

$$\Delta y' = \Delta y \quad (1c)$$

$$\Delta z' = \Delta z. \quad (1d)$$

In this set of equations, the variables $\Delta x'$, Δx, $\Delta y'$, Δy, $\Delta z'$, Δz refer to distances traveled by an object in each of the three orthogonal spatial dimensions, whereas $\Delta t'$ and Δt are the corresponding elapsed times for the same motion as measured by two different observers who are moving at constant speed v relative to one another along a common x,x' axis of the coordinate system. It is clear that this transformation is inconsistent with the above relation from the Fresnel light-drag extrapolation. Instead, the latter would indicate that c(v) = c + v if the light moves along the x'x' axis, for example, not simply c.

Voigt's Space-time Mixing Conjecture

Belief in the Galilean transformation remained strong and attempts to explain the discrepancy over a period of many decades were based on the assumption of the existence of an object commonly referred to as an "aether," which supposedly had similar properties for light as does the rest frame of sound waves [3]. Voigt [2] deviated from this view by speculating that the real problem lay in the Galilean transformation itself. He proposed that a change be made in eq. (1a) which involved the elimination of the centuries-old belief that space and time are completely separate entities. He added a term which mixed the spatial and time variables in such a way as to guarantee satisfaction of the light-speed constancy condition indicated by experiment. In so doing, he left eq. (1b) untouched, but was forced to alter eqs. (1c) and (1d) for transverse motion of the light waves in order to make the equality hold for all directions. The result is eqs. (2a-d):

$$\Delta t' = \Delta t - vc^{-2}\Delta x = \eta^{-1}\Delta t \quad (2a)$$

$$\Delta x' = \Delta x - v\Delta t \quad (2b)$$

$$\Delta y' = \gamma^{-1}\Delta y \quad (2c)$$

$$\Delta z' = \gamma^{-1}\Delta z. \quad (2d)$$

In this set of equations, $\gamma = (1-v^2c^{-2})^{-0.5}$ and $\eta = \left(1 - vc^{-2}\frac{\Delta x}{\Delta t}\right)^{-1}$.

The main problem with the VT is that it is not consistent with Galileo's Relativity Principle (RP). This can be seen most easily by inverting eqs. (2c) and (2d). The result should be the same as if one simply interchanged observers by exchanging the primed and unprimed symbols and reversing the sign of v (a procedure which is referred to as Galilean inversion in the following discussion), thereby simulating a role-reversal in obtaining the measured results. The required inversion does not occur in this way (note that changing the sign of v has no effect on the value of γ).

Lorentz Transformation

Larmor [4,5] apparently noticed this deficiency and proposed the alternative set of equations given below:

$$\Delta t' = \gamma(\Delta t - vc^{-2}\Delta x) = \gamma\, \eta^{-1}\Delta t \quad (3a)$$

$$\Delta x' = \gamma\, (\Delta x - v\Delta t) \quad (3b)$$

$$\Delta y' = \Delta y \quad (3c)$$

$$\Delta z' = \Delta z \quad (3d)$$

It eliminates the problem with Galilean inversion, as is obvious from its eqs. (3c-3d). At the same time, this transformation also satisfies the light-speed constancy condition. This result is evident from forming the following linear combination of the squares of its variables:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 \quad (4)$$

The latter equation has come to be known as Lorentz invariance. It clearly shows that if the measured speed of light is c in one frame, it must also have the same value in the other frame.

Lorentz came up with exactly the same set of equations shortly after Larmor did [5]. They have since been referred to as the Lorentz transformation (LT), despite Larmor's priority in their formulation. Lorentz also pointed out [6] that there was a degree of freedom in developing a space-time transformation which satisfies the light-speed constancy condition. One can multiply each of the four relations in eqs. (3a-d) of the LT by a common factor ε on the respective right-hand sides without affecting this characteristic. This procedure results in adding a factor of ε^2 on the right-hand side of eq. (4), thereby clearly preserving light-speed constancy in the process. The Voigt transformation (VT) of eqs. (2a-d) is obtained from eqs. (3a-d) by setting ε equal to γ^{-1}, for example. This result underscores the fact that not every choice of ε is also consistent with the RP, however.

Clock Puzzle

There is nonetheless also a problem with the LT [7-9]. This is because it makes two predictions which are incompatible with one another, namely proportional time dilation and remote non-simultaneity. Both effects are discussed in Einstein's landmark 1905 paper [10]. In the former case, it is proven on the basis of the LT that the rates of clocks in different inertial systems, and therefore time differences Δt' and Δt measured with them, are strictly proportional to one another, i.e. $\Delta t' = \Delta t/X$, whereby X is constant as long as no change in the velocities of either rest frame occurs. At the same time, it is clear from eq. (3a) that if $\Delta t = 0$, corresponding to *simultaneous* observation, and both v and Δx are not equal to zero, the result is $\Delta t' \neq 0$, i.e. *non-simultaneous* observation. This characteristic of the LT, referred to as remote non-simultaneity (RNS), is clearly inconsistent with proportional time dilation since it violates the axiom of elementary algebra that states that multiplication of any finite number, in this case X or 1/X, with zero must result in a value of zero. This contradiction has been referred to as the "Clock Puzzle" [9], and it therefore unequivocally rules out the LT as a physically valid space-time transformation.

The above analysis shows clearly that it is eq. (3a) of the LT which is directly responsible for the RNS prediction. Furthermore, it also is responsible for the prediction of proportional time dilation [11]. The same two conclusions follow directly from eq. (2a) of the VT.

The unique role of either eq. (2a) or eq. (3a) in reaching these conclusions shows that the cause of the discrepancy between RNS and proportional time dilation *is the space-time mixing characteristic of both the LT and VT.*

Newtonian Simultaneity

In order to obtain more insight into this point, it is

helpful to recall that the object of the measurements described in the various space-time transformations under discussion is always assumed to be an "inertial system," i.e. one that is not subject to any unbalanced external force. Newton's First Law of Motion states that any such object must move with constant speed and direction until some external force is applied to it.

The question thus arises as to the way the various properties of an inertial object vary over time. In the absence of any unbalanced external force, it would seem entirely consistent with the First Law to assume that they all remain constant as well (Law of Causality) and that this conclusion applies in particular to *the rates of inertial clocks*. Just as the First Law does not imply that the velocities of different inertial objects must all be the same, however, it is equally clear that the rates of two different inertial clocks also may differ. Since both rates are constant, however, one can safely conclude on this basis that *their ratio must also be constant.* As a consequence, elapsed times $\Delta t'$ and Δt for the same pair of events measured with two such inertial clocks must also adhere to a strict proportionality, *i.e.* $\Delta t'=Q^{-1}\Delta t$, where Q is the above ratio of clock rates. This proportionality may appropriately be referred to as "Newtonian Simultaneity" because of its close relation to Newton's First Law of Motion as well as his well-known insistence that any event throughout the universe must occur at the same time for all observers. The latter relationship is the obvious consequence of the above proportionality of inertial clock rates, since it does not allow for one measured time difference to be equal to zero without the other being so as well.

Newton-Voigt Transformation

But is Newtonian Simultaneity compatible with light-speed constancy? The answer is clearly "yes," by virtue of the degree of freedom pointed out by Lorentz [6] with regard to space-time transformations which fulfill the latter condition. One can either multiply each of right-hand sides of the four equations of the VT with a factor of ηQ^{-1} or do the same using a factor of $\eta(\gamma Q)^{-1}$ for each of the four LT equations [7,12] . In both cases, the result is:

$$\Delta t' = \eta Q^{-1}(\Delta t - vc^{-2}\Delta x) =\eta (\eta Q)^{-1}\Delta t = Q^{-1}\Delta t \quad (5a)$$

$$\Delta x'= \eta Q^{-1} (\Delta x-v\Delta t) \quad (5b)$$

$$\Delta y'= \eta(\gamma Q)^{-1} \Delta y \quad (5c)$$

$$\Delta z'= \eta(\gamma Q)^{-1} \Delta z \quad (5d)$$

The above transformation contains the Newtonian Simultaneity condition explicitly in its eq. (5a), while also adhering to exactly the same light-speed constancy inherent in the VT and LT. The latter relationship is made explicit in the equation given below,

$$\Delta x'^2+\Delta y'^2+\Delta z'^2-c^2\Delta t'^2=\eta^2(\gamma Q)^{-2} (\Delta x^2+\Delta y^2+\Delta z^2-c^2\Delta t^2) \quad (6)$$

which is obtained by forming a linear combination of the squares of each of the four variables on the left-hand sides of eqs. (5a-d), similarly as is done for the LT equations in arriving at the Lorentz invariance condition of eq. (4).

It remains to be shown that the NVT also satisfies Galileo's RP. This is done most easily by applying the Galilean inversion procedure to each of the four NVT equations. The result is the corresponding inverse relation of the NVT, as required by the RP. This procedure places a condition on the quantity Q' which occurs in the inverse of eq. (5a), namely QQ'=1. Recalling the original definition of Q as the ratio of clock rates as measured by one observer, it is clear that its counterpart Q' is simply the reciprocal of

the original ratio as viewed from the standpoint of the other observer. The QQ'=1 relationship thus arises in a perfectly consistent manner to that found between forward and reverse conversion factors in any situation where different units are employed in expressing the results of a given measurement. The other condition that needs to be satisfied arises in a perfectly straightforward manner without the necessity of introducing any new conditions, but rather insisting that the η' factor in the inverse set of equations be formed directly by applying Galilean inversion to the original η factor which appears in eqs. (5b-d). The result is thus $\eta' = (1+vc^{-2}\Delta x'/\Delta t')$. Applying Galilean inversion to eq. (5b) leads to the corresponding inverse relationship, for example, upon making use of the following identity: $\eta'\eta = \gamma^2 QQ' = \gamma^2$, which follows directly from the above definitions [11]. In the following, eqs. (5a-d) will be referred to as the Newton-Voigt transformation (NVT). It satisfies both of Einstein's postulates of relativity [10], namely consistency with Galileo's RP and the Voigt light-speed constancy conjecture [2], while at the same time avoiding any violation of Newtonian Simultaneity.

Hafele-Keating Atomic Clock Measurements

In order to apply the NVT for a given pair of rest frames, it is necessary to know the corresponding value of the parameter Q. Experimental data are clearly required to make this determination. An important step in achieving this goal on a general basis was made by Hafele and Keating in 1971 [14,15]. They studied the rates of atomic clocks that were carried onboard aircraft as they circumnavigated the globe in opposite directions. They found that the eastward flying clocks returned to the airport of origin with significantly less elapsed time than those left behind there, whereas those flying in the westward direction returned with more elapsed time than the latter. Quantitative examination of the timing results indicated that the rates of clocks moving at the same altitude were inversely proportional to γ (v), where v is the speed of the clocks relative to the earth's center of mass (ECM). On this basis, it can be concluded that the following relation holds for the elapsed times Δt' and Δt of two such clocks traveling with respective speeds v and v' relative to the ECM:

$$\Delta t' \gamma (v') = \Delta t \gamma (v) \quad (7)$$

A correction for gravitational effects on the clock rates was made by using formula for the gravitational red shift introduced in 1907 [16].

It also can be noted that eq. (7) holds quantitatively in describing the periods of x-ray sources and absorbers mounted on a high-speed rotor of radius R moving with rotational frequency ω [17-19], in which case the speeds v = Rω and v' = R'ω must be taken relative to the rest frame of the rotor axis itself. It also is consistent with Einstein's conjecture [10] with regard to the rate of a clock moving with an electron in a closed path, in which case the corresponding speeds are taken relative to the rest frame in which the force was applied to the electron. Einstein also gave an example in which the elapsed time of a clock located at the Equator is compared with that of its counterpart located at one of the earth's Poles, in which case the speeds to be inserted in eq. (7) are again taken relative to the ECM.

Universal Time-dilation Law

In view of the commonality of the above experimental results, it is appropriate to refer to eq. (7) as the Universal Time-dilation Law (UTDL) [20]. In order to apply it in a given case, it is necessary to define a rest frame from

which to compute the speeds in the γ factors, and this has been designated in previous work [21] as the Objective Rest System (ORS). In the present context of the NVT, the most interesting point is that the UTDL can be used directly to obtain the value of the parameter Q. Comparison with eq. (5a) shows that

$$Q = \gamma(v')/\gamma(v) \quad (8)$$

Moreover, application of Galilean inversion leads to the conclusion that the corresponding value of Q' in the inverse NVT is just the reciprocal of Q, as required by the RP.

Global Positioning Navigation System

The time-dilation experiments carried out by Hafele and Keating [14,15] had a great impact on the development of the Global Positioning System (GPS). A key requirement is the capacity to measure the elapsed time Δt for light signals to pass between an orbiting satellite and a position on the earth's surface. The corresponding distance is then determined to be cΔt. To obtain maximum accuracy, it is necessary to insure that the *atomic clock on the satellite runs at the same rate as its counterpart on the ground.* To this end, a "pre-correction" is made to the frequency of the satellite clock prior to launch into a known orbit around the earth. The amount of this correction is determined [22-24] on the basis of the UTDL of eq. (7) along with the predicted gravitational red shift. The accuracy of the GPS distance measurements in everyday applications thus serves as a strong confirmation of the UTDL. The procedure also demonstrates the reliability of eq. (5a) of the NVT, since it would be pointless to make the clock-rate adjustment if events do not occur simultaneously for an observer on the satellite and his counterpart on the ground.

Relativistic Conversion Factors for Physical Properties

A particularly insightful way of looking at the Q parameter in the NVT is as a "conversion factor" between different units of time. Given an elapsed time measured on the satellite clock, it allows one to determine the corresponding value that would be obtained using the earth-based counterpart. The situation is wholly similar to what occurs when one changes from m to cm in distance determinations or from lb to kg in weighing a given object. A confusing aspect in the present case is that the observer located in any one rest frame is *employing the same set of standard units as someone who is moving with respect to him* at any given gravitational potential. The actual distinction in their respective units can only be ascertained by mutual observation of the properties of the same object or event. In absolute terms, each such result will be the same for both observers, but the corresponding numerical value will be different simply because they employ different units in which to express their result. The fact that observers in different rest frames are unable to establish such a difference based on exclusively *in situ* measurements is a direct consequence of the RP. It would be a clear violation of the latter if there were any way to recognize such a change in standard units simply by moving from one inertial rest frame or altitude to another.

The above discussion applies to other physical properties as well. For example, the light-speed constancy condition indicates that the unit of speed/velocity is the same in all rest frames, i.e. which are also located at the same gravitational potential. Otherwise, one cannot explain on an objective basis how the speed of a light pulse relative to its source has the same value of c for all observers located at the same gravitational potential. As a consequence, it can also be concluded that the conversion factor for distance is exactly the same as for elapsed times, i.e. it also has a value of Q. One knows, for example, that the wavelength of light changes in direct proportion to the period of the

radiation as the light source is accelerated. Experimental results [25] for the variation of the inertial mass of an electron upon acceleration indicate that it changes in direct proportion to the lifetimes of meta-stable particles. Hence, the conversion factor for inertial mass also has the same value of Q as for elapsed times and distances. Moreover, once these factors have been determined, it is clear that the corresponding conversion factor for any other physical property can be determined on the basis of its composition in terms of these three fundamental quantities, i.e. distance, inertial mass and time. Each such value must accordingly be an integral power of Q [26,27]. Ultimately, these relationships stem from the fact that the NVT is consistent with the principle of objectivity of the measurement process, something which cannot be said for the LT.

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Conflict of Interest

None

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